

Subtask 1

The lines with the minimal and maximal slope will be the ones drawn from the top of the left segment to the bottom of the right segment and the one drawn from the bottom of the left segment to the top of the right segment.

Subtask 2

This subtask is meant to reward noticing that the valid values of m form a continuous interval, a rigorous proof of this will be done later in the editorial.

To solve this subtask you need to binary search for the lowest m and highest m , with your left or right limit in the binary search being 1.

To check if a value m is valid we need to know if there exists some b such that the line $mx + b$ goes through every segment. For some interval L_i, R_i the range of possible values for b can be calculated as follows:

$$L_i \leq m \cdot x_i + b \leq R_i$$

$$L_i - x_i \cdot m \leq b \leq R_i - x_i \cdot m$$

Now, we have the intervals of valid b for a single interval. If these intervals intersect for all segments, that means there is a value of b that goes through every segment and that means that the slope m is valid.

Subtask 3

Notice that a line with minimal/maximal slope will always be touching at least two segment endpoints.

We prove this by contradiction. Say a minimal/maximal slope is touching at most one point. We can rotate around the point it is touching, or an arbitrary one if it is not touching any point. We rotate it until it touches another point and we will still have a valid line with smaller/bigger slope.

This fact gives us a $O(n^3)$ solution, where we try all lines generated by drawing a line between two endpoints of segments and then check validity using the procedure from subtask 2.

Subtask 4

Now we sort the intervals by x . So if $i < j$ then $x_i < x_j$.

Let's revisit the intervals of valid values of b . As we said earlier, an m is valid if all of the intervals $[L_i - x_i \cdot m, R_i - x_i \cdot m]$ intersect, i.e. there exists a b that belongs to every one of these intervals.

For a pair of segments i and j such that $i < j$ we can calculate for which m their intervals intersect. We need to satisfy these two conditions:

$$L_i - x_i \cdot m \leq R_j - x_j \cdot m$$

and

$$R_i - x_i \cdot m \geq L_j - x_j \cdot m$$

Working out these inequalities we get :

$$\frac{L_j - R_i}{x_j - x_i} \leq m \leq \frac{R_j - L_i}{x_j - x_i}$$

Now we have an interval for a pair of segments. When we merge these intervals for every pair we get the interval of all valid values of m . If these intervals do not intersect, then there is no solution.

How do we know that by taking the intersection of this we preserve the fact that there exists a common b for each m in the interval? For these m we know that the intervals for all valid b values intersects pairwise by construction, but we are missing the fact that they all have a common intersection.

Now we want to prove that if n intervals intersect pairwise then they have a common point. Take the highest left bound of all the n intervals and the lowest right bound. Call them L and R . Since everyone intersects pairwise it must be that $L < R$, so the interval $[L, R]$ belongs to all of the n original intervals.

So we conclude that the intersection of pairwise valid values of m represents the range of valid values for all segments. This also represents a formal proof that the range of values of m is continuous.

Subtask 5

From what we said earlier we can deduce that $m \geq m_{min}$ if for every pair of i, j s.t. $i < j$ the following is true:

$$R_i - x_i \cdot m \geq L_j - x_j \cdot m$$

Note that this is the inequality from which we got the leftpoints of the m intervals. This is equivalent to saying that for every j , $\min_{i < j} (R_i - x_i \cdot m) \geq L_j - x_j \cdot m$

Since this condition is monotonic (if it holds for m it works for any m' greater than m), we can binary search for the minimum m .

From here we can either do something similar to find the maximum or binary search for the maximum like in subtask 3, yielding an overall complexity of $O(n \cdot \log(\text{precision of } m))$