Rabbit - Analysis

Many approaches are possible for partial scores. Here we will describe only approaches that yield full score, as those are the interesting ones.

To build some intuition about the strategies we can use, let's consider how the set of possible rabbit locations changes after each cell check. In particular, we will focus on how the number of possible locations changes.

Checking a cell when the rabbit is *curious*

- If K was a possible rabbit location, the possible locations would decrease by 1.
- If $K - 1$ **and** $K + 1$ were both possible rabbit locations, the possible locations will decrease by 1 (since those two are "merged" in K)

In total, checking a cell when the rabbit is curious will reduce the possible locations by 0, 1, or 2.

Checking a cell when the rabbit is *scared*

- If K was a possible rabbit location, the possible locations would decrease by 1.
- If 1 **and** 2 were both possible rabbit locations and $K > 2$, the possible locations will decrease by 1 (those two get "merged" in 1)
- If N **and** $N - 1$ were both possible rabbit locations and $K < N - 1$, the possible locations will decrease by 1 (those two get "merged" in N)

In total, checking a cell when the rabbit is scared will reduce the possible locations by 0, 1, 2 or 3.

Observing these two cases, we can build some intuition on what kind of checks are profitable. Clearly, when the rabbit is curious, we want to keep the possible locations in a contiguous interval and check cells within that interval, and when the rabbit is scared, we would like to have possible locations next to both edges, which requires at least two intervals of possible locations. The issue is that these conflict with each other, and furthermore whenever the rabbit is curious, the possible locations are dragged away from the edges, making it impossible to maximize the gains from both moods simultaneously. Thus. we have split the solution in two strategies

Solution for $C \geq S$

We keep the possible locations in an interval $[L, R]$. Initially, that's the interval $[1, N]$. We employ the following strategy:

- When the rabbit is curious, check cell $L + 1$. The new interval of possibilities becomes $[L + 1, R - 1]$. The amount is decreased by 2.
- When the rabbit is scared, check cell L. The new interval of possibilities becomes $[L + 1, \min(N, R + 1)]$. The amount is decreased by 1 if $R < N$ and 2 if $R = N$.

With this strategy we will start out reducing the amount by 2 in the initial S scared seconds. However, once a full cycle of $S + C$ seconds passes, the C curious seconds will have pulled the interval C leftwards.

The subsequent S scared seconds won't be enough to push it back to the right corner since $C \geq S$, so in the long run we can presume the scared checks will reduce the amount by 1.

For $S \leq C \ll N$ the number of possible locations decreases by $(2C + S)$ every $S + C$ seconds. Hence, we will need roughly $\frac{N(S+C)}{S+2C}$ seconds. Since $C \geq S$, we have $T = \frac{N(S+C)}{S+2\max(S,C)} + 3\max(S,C) = \frac{N(S+C)}{S+2C} + 3C$. Clearly, this yields full score.

Solution for $S \geq C$

Since $S \geq C$ here, it is a better idea to keep possible locations next to both edges, in order to take advantage of the full power of scared rabbit checks. We keep the possibilities in the following intervals: $[L, R) \cup [X, N]$. Initially, we have $\left[1, \frac{N(S-C)}{3S}\right) \cup \left[\frac{N(S-C)}{3S}, N\right]$. Then the strategy is:

- When the rabbit is curious, check cell N. The new possibilities are $[L+1, R+1) \cup [X+1, N]$. The amount is decreased by 1. Note that this decrease happens on the right interval.
- When the rabbit is scared, check cell X. The new possibilities are $[\max(1, L-1), R-1) \cup [X+2, N]$. The amount is reduced by 2 if $L > 1$ and 3 if $L = 1$. Note that a decrease of 2 always happens on the right interval and a decrease of 0 or 1 happens on the left one.

With this strategy we will be reducing by 3 for the first S scared seconds. The next C curious seconds will reduce by 1 and shift the left interval C rightwards. Then for C scared seconds we will be reducing by 2 since we will have $L > 1$, but for the next $S - C$ we will again by reducing by 3.

Thus, for $C \leq S \ll N$ the size of the right interval decreases by $2S + C$ every $S + C$ seconds and the size of the left interval decreases by $S - C$ every $S + C$ seconds. Thus, the two intervals will shrink to size 0 at (approximately) the same time, due to the chosen ratio between their initial sizes. The total decrease is $2S + C + S - C = 3S$ for every $S + C$ seconds. Hence we will need roughly $\frac{N(S+C)}{3S}$ seconds. Since $S \geq C$, we have $T = \frac{N(S+C)}{S+2\max(S,C)} + 3\max(S,C) = \frac{N(S+C)}{3S} + 3S$. Clearly, this yields full score.

Note that it can be proven that the above two strategies are asymptotically optimal for their respective cases using an amortized analysis with potentials. Futhermore, for $S = C$ both strategies give the same asymptotic results for large N and full score.

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