

## A standard problem Solution Momchil Ivanov

Given the problem constraints one can guess that the answer for each possible range of rows  $(i, j)$ ,  $(1 \leq i \leq j \leq N)$  should be precomputed before the queries, so that we can answer each of them with  $O(1)$  complexity.

### Precomputing:

We would start processing the rows from top to bottom and for each chosen  $j$  ( $j$  would be the upper row bound, meaning the bigger) we would choose all possible lower bounds  $1 \leq i \leq j$ , starting from the top again.

Let  $dp[i][j]$  be the area of the biggest table we can fit between rows  $[i, j]$ , then:

- ▲  $dp[i][j] = \max(dp[i][j-1], \max(dp[i+1][j], A_{j-i+1}))$ , where  $A_{j-i+1}$  is the area of the biggest table that fits between  $[i, j]$  doesn't contain ones and has height  $(j-i+1)$ .

Given the order in which we are processing the ranges it follows that if we are computing  $dp[i][j]$ , then we would have information for  $dp[i][j-1]$  already but not for  $dp[i+1][j]$ . This means that while we are processing all possible lower bounds ( $i$  from 1 to  $j$ ) we would be computing  $dp[i][j]$  with the formula:

- ▲  $dp[i][j] = \max(dp[i][j-1], A_{j-i+1})$

Once we have traversed all possible lower bounds  $i$  from top to bottom, we would traverse them again, but from bottom to top (i.e. starting from  $j$ , going to 1) in order to compare our current optimal result for  $dp[i][j]$  with  $dp[i+1][j]$ , i.e. for each  $i$  we do:

- ▲  $dp[i][j] = \max(dp[i][j], dp[i+1][j])$

The only thing that is left is to compute  $A_{j-i+1}$  for all  $i$ , where  $1 \leq i \leq j$ . In other words we need to compute the biggest tables of height in the range  $[1, (j-i+1)]$  that are lying on row  $j$ .

### Computing $A_{j-i+1}$ :

For each column of row  $j$  we would find how much can we go up until we reach a 1, let  $h[j]$  be the number of rows we can go up like this, including the row  $j$ . For example, given the table:

```
0 1 0 0
1 0 0 0
0 0 0 0
```

For the last row we have the following values of  $h$ :  $h[1] = 1$ ,  $h[2] = 2$ ,  $h[3] = h[4] = 3$ . For each  $h[k]$  we would compute the minimum  $p$ , where  $p \leq k$  such that  $h[1] \geq h[k]$ , for all  $l$  in  $[p, k]$ . Let  $Left[k] = k - p + 1$ , we define similarly  $Right[k] = p - k + 1$ , where  $p$  is the maximum  $p$  ( $p \geq k$ ) such that  $h[1] \geq h[k]$ , for all  $l$ , which are in  $[k, p]$ .

In the given example we have  $Left = \{1, 1, 1, 2\}$ ,  $Right = \{4, 3, 2, 1\}$ . Both  $Left[]$  and  $Right[]$  can be precomputed (separately) in  $O(n)$  complexity, using a stack structure. In it we would keep pairs  $(h, w)$ , such that if one pair  $(h_1, w_1)$  is on the top of another pair  $(h_2, w_2)$  in the stack then  $h_1$  should be bigger than  $h_2$ . This is how we would use such a stack to compute  $Left[]$ , for example:

```

sz = -1;

for( int j = 1; j <= m; ++j ){

    sum = 0;

    while( sz >= 0 ){

        if( st[sz].first >= h[j] ) {sum += st[sz].second; sz--;}

        else break;

    }

    st[++sz] = make_pair(h[j], sum + 1);

    Left[j] = sum + 1;

}

```

In the above C++ code  $h[]$  and  $Left[]$  are as explained above and  $st[]$  is an array of pairs used as a stack.

In a similar way can be computed  $Right[j]$  if we simply loop from  $j = m$  to 1.

Given  $Left[]$  and  $Right[]$  we want to compute  $A[len]$ , where  $1 \leq len \leq (j - i + 1)$ , and  $A[len]$  gives the maximum width of a table with height  $len$  that does not contains ones inside and lies on the  $j^{\text{th}}$  row. It is not difficult to see that  $A[len] =$

$\max(A[\text{len}+1], \max(\text{Left}[k] + \text{Right}[k] - 1))$ , where  $\max(\text{Left}[k]+\text{Right}[k]-1)$  is the maximum such sum for which  $h[k] = \text{len}$ .

Now that we have computed  $A[]$ ,  $A_{j-i+1}$  is simply equal to  $A[j-i+1]*(j-i+1)$ .

Given the above analysis the solution of this problem should have a complexity of  $O(N*M)$ . For clarifications please check the author's solution - standard.cpp.