# **Desired Solution**

We begin by reformulating the problem. Let  $pre_i = \sum_{j=1}^i d_j$ . For each position  $1 \le i \le n$ , define S(i) as the number of stars Ian can collect if he starts at position i. Then:  $S(i) = |\{j \ge i \mid t_j \le pre_j - pre_{i-1}\}|$ 

There are two neat approaches to solving this problem efficiently, both based on computing all values of S(i).

#### Full Solution 1

We modify the condition, note that  $t_j \leq pre_j - pre_{i-1} \iff pre_{i-1} \leq pre_j - t_j$ . We define  $a_j = pre_j - t_j$ , then the formula for S(i) becomes  $S(i) = |\{j \geq i \mid a_j \geq pre_{i-1}\}|$ . We compute S(i) by enumerating i from right to left. We maintain a multiset that stores all current values of  $a_j$  for  $j \geq i$ . For each i, we first insert  $a_i$ , then we compute how many elements in the multiset are  $i \geq pre_{i-1}$ .

This yields an  $O(N \log N)$  solution using either a Binary Indexed Tree (Fenwick Tree) or an ordered set (e.g., \_\_gnu\_pbds::tree in C++). Time Complexity:  $O(N \log N)$ 

### **Full Solution 2**

Using the same simplification  $S(i) = |\{j \ge i \mid a_j \ge pre_{i-1}\}|$ , we now observe that if there exists j < i such that  $pre_{j-1} \le pre_{i-1}$ , then starting from i is never optimal. This means that we only need to consider starting positions i such that  $pre_{i-1}$  is the minimum in the prefix, we call these valid starting positions.

We compute S(i) for these valid starting positions by sorting all  $a_j$  in non-increasing order. Then we iterate over all valid starting positions i in increasing order. We maintain a set A of indices  $j \geq i$  such that  $a_j \geq pre_{i-1}$ . We remove any indices j < i in A, since they are no longer valid. After the cleanup, S(i) = |A|. We can use a priority queue or a multiset to maintain and update A dynamically. Time Complexity:  $O(N \log N)$ 

# Subtask 1 $(N \leq 2000)$

These subtasks are intended for brute-force solutions. For each starting point i, simulate and count the amount of stars Ian can collect. Time Complexity: $O(N^2)$ 

## Subtask 2 (at most 20 negative $d_i$ 's)

This subtask supports Solution 2. Since at most 20 prefix sums may decrease (due to negative values), the number of valid starting positions is at most 20. By identifying all valid starting positions and simulate each in O(N), we obtain a solution to pass this subtask. Time Complexity:  $O(N|\{i: P_i < 0\}|)$ 

## Subtask 3 (no positive $d_i$ )

Since all  $d_i \leq 0$ ,  $pre_i$  is a non-increasing sequence. We can use binary search to count the number of indices which satisfy the inequality  $a_i \geq pre_{i-1}$  as described in the full solutions. Time Complexity  $O(N \log N)$ 

### Subtask 4

This subtask is intended to encourage a simplified version of Solution 1. Since the number of stars collected is at most 20, we can calculate the amount of elements in the multiset that are  $\geq pre_{i-1}$  by maintain the top 20 largest elements in a fixed-size array, using std::set and prev() function, or use a priority\_queue and pop elements greater than  $pre_{i-1}$  until none remain. Time Complexity:  $O(N \log N + N \times \text{ans})$  or  $O(N \log N \times \text{ans})$  depending on which data structure is used